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# Dynamic economic modeling of intertemporal consumption with memory using Volterra integral equations

## *Modelado económico dinámico del consumo intertemporal con memoria mediante ecuaciones integrales de Volterra*

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## ABSTRACT

The study of intertemporal consumption with memory represents a significant advance in understanding modern economic dynamics, where agents' current decisions depend not only on present income but also on their past income history. The purpose of this research is to develop a dynamic economic model of intertemporal consumption with memory, formulated through Volterra integral equations, which allow an accurate representation of the influence of the past on current consumption behavior. The methodology was structured in four main stages: theoretical formulation of the mathematical model with memory through an exponential kernel; computational implementation in Python for numerical resolution using discrete convolution; simulation and visualization of the results under different economic scenarios; and validation of the obtained solutions through analytical and numerical comparison. The main results show that consumption $(C(t))$ gradually adjusts to income as a function of the memory parameter $(\alpha)$ and the income growth rate $(\beta)$. In particular, it is shown that higher values of $(\alpha)$ cause the influence of past income to decay more rapidly, while a $(\beta > 0)$ leads to higher equilibrium levels of consumption. In conclusion,

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the proposed model provides a coherent and quantitative representation of consumption with memory, integrating theoretical and computational elements that broaden the understanding of the intertemporal behavior of economic agents. Moreover, the applied methodology demonstrates the usefulness of Volterra equations as a versatile tool in dynamic economic analysis and in the simulation of past-dependent phenomena.

**Keywords:** Intertemporal consumption, Economic memory, Volterra integral equations, Dynamic modeling, Computational simulation.

**RESUMEN**

El estudio del consumo intertemporal con memoria representa un avance significativo en la comprensión de la dinámica económica moderna, donde las decisiones actuales de los agentes no dependen únicamente del ingreso presente, sino también del historial de ingresos pasados. Esta investigación tiene como propósito desarrollar un modelo económico dinámico del consumo intertemporal con memoria, formulado mediante ecuaciones integrales de Volterra, las cuales permiten representar de manera precisa la influencia del pasado en el comportamiento actual de consumo. La metodología se estructuró en cuatro etapas principales: Formulación teórica del modelo matemático con memoria mediante un kernel exponencial; Implementación computacional en Python para la resolución numérica mediante convolución discreta; Simulación y visualización de los resultados bajo distintos escenarios económicos; y Validación de las soluciones obtenidas mediante comparación analítica y numérica. Los principales resultados muestran que el consumo $C(t)$ se ajusta gradualmente al ingreso en función del parámetro de memoria $\alpha$ y de la tasa de crecimiento del ingreso $\beta$. En particular, se demuestra que, a mayor valor de $\alpha$, la influencia del ingreso pasado disminuye más rápidamente, mientras que un $\beta > 0$ conduce a niveles de consumo de equilibrio más altos. En conclusión, el modelo propuesto ofrece una representación coherente y cuantitativa del consumo con memoria, integrando elementos teóricos y computacionales que amplían la comprensión del comportamiento intertemporal de los agentes económicos. Además, la metodología aplicada demuestra la utilidad de las ecuaciones de Volterra como herramienta versátil en el análisis económico dinámico y en la simulación de fenómenos dependientes del pasado.

**Palabras clave:** Consumo intertemporal, Memoria económica, Ecuaciones integrales de Volterra, Modelado dinámico, Simulación computacional.

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## INTRODUCTION

Dynamic economic models provide a mathematical framework for examining how economic variables change over time. In the case of intertemporal consumption, household decisions are influenced not only by current income but also by previous economic conditions. Past income may affect consumption habits, expectations, and the way individuals adjust their spending when their financial circumstances change. Consequently, incorporating historical information into consumption models is relevant for understanding the persistence of economic behavior.

One way to represent this dependence on the past is through mathematical models with memory. Although differential equations with delays can describe certain forms of temporal dependence, integral formulations offer an alternative in which previous observations contribute to the current state of a variable. In particular, Volterra integral operators allow historical information to be incorporated through a kernel function that assigns different weights to past values. This property makes them suitable for examining consumption processes in which previous income continues to influence present decisions.

In this regard, the scientific literature highlights significant studies concerning mathematical modeling applied to economics. Among the influential authors in this field, the following stand out: first, Villegas et al. (2024) conducted mathematical economics research through a study of mathematical models applied to economics; second, Wu (2014) addressed the predator-prey model and its applications to economics; third, Gandolfo (2007) studied Lotka-Volterra equations in economics as an Italian precursor; fourth, Roa et al. (2004) analyzed economic growth models and complex dynamics; and fifth, Devi et al. (2026) developed an overview of integral equations and their applications to economic disciplines.

The present study focuses on the application of a Volterra convolution integral operator to intertemporal consumption with memory. Its main objective is to formulate a dynamic consumption model in which historical income influences current consumption, derive an analytical solution for an exponential memory kernel and an exponential income function, and examine the resulting behavior through numerical simulations. The analysis emphasizes the role of the memory parameter and the income growth rate in determining consumption adjustment over time.

For the mathematical formulation, an exponential kernel is selected to represent the gradual reduction in the influence of past income. This choice permits an explicit analytical solution and provides a direct interpretation of the memory parameter. The theoretical development is complemented by a computational implementation in Python, where discrete convolution is used to examine consumption trajectories under different parameter values.

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The numerical analysis considers a synthetic income series covering 360 monthly periods. This extended time horizon allows the behavior of consumption to be examined as the influence of historical income changes. Additional illustrative scenarios involving inflation, savings, and debt are considered to explore how different economic conditions may affect the simulated trajectories. These scenarios are interpreted within the assumptions and limitations of the computational model.

The contribution of this research lies in connecting an explicit mathematical representation of economic memory with its analytical solution and computational illustration. Rather than assuming that consumption responds instantaneously to income, the proposed formulation incorporates the accumulated influence of past income through a memory kernel. This approach provides a basis for analyzing consumption adjustment and for developing future extensions involving additional economic variables.

## Volterra integral equations and economic model

### Volterra integral equations

Volterra integral equations are a particular class of equations of the following form:

$$x(t) = f(t) + \int_a^t K\left(t, s, x(s)\right) ds, \qquad (1)$$

where $x(t)$ is the unknown function, $f(t)$ is a known function, and $K\left(t, s, x(s)\right)$ is the kernel of the equation, which may depend on the current time $t$, the integration time $s$, and the function $x(s)$. Equation **(1)** corresponds to a Volterra integral equation of the second kind. Its linear or nonlinear character depends on how the unknown function appears in the kernel. Regarding equation **(1)**, there are two main types: linear (when $K(t, s, x) = K(t, s)x(s)$) and nonlinear (when $K$ depends on $x(s)$ in a nonlinear manner). In both cases, these equations are widely used in physics, biology, and engineering to model systems with memory or delayed dependence Carrasco et al. (2021) and Zhang et al. (2018).

### Some solution methods for Volterra equations

Volterra equations rarely possess exact closed-form solutions; consequently, analytical or numerical approximation methods are employed Brunner (2004). Among the most widely used methods are: *The Adomian decomposition method*, which enables the construction of an approximate solution in the form of a series by splitting the equation into linear and nonlinear components Jaafar (2025) and Rahman (2007). And, *The series solution method*, which constructs an approximation via a power series expansion Noeiaghdam et al. (2024) and Yuan & Liang (2026). The variational iteration

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method, which relies on functional formulations to refine successive approximations Wazwaz (2015) and Goyal & Sharma (2017). In the present study, the analytical solution is obtained through direct integration, whereas the numerical implementation uses discrete convolution. The Adomian decomposition method is discussed as a possible alternative for nonlinear extensions of the proposed model.

**Memory in economic models**

In an economic context, many phenomena depend not only on the current state but also on their past trajectory. For example, consumption habits, consumer confidence, or inflation expectations can be influenced by past events Lukač (2026) and Kalmanovitz (2001). The definition of *consumption with memory* implies that current consumption $C(t)$ depends not only on present income $Y(t)$ but also on a weighted aggregation of past incomes. This idea can be modeled using a Volterra integral equation of the following type:

$$C(t) = \int_0^t K\,(t-s)Y(s)\,ds, \qquad (2)$$

where $K(t-s)$ is the memory function representing the weight assigned to past income in the current consumption decision. A common example of a kernel is the exponential function $K(t-s) = \alpha e^{-\alpha(t-s)}$, which assigns greater importance to recent income and less to more distant income. This type of formulation is consistent with theories such as the permanent income hypothesis or consumption habits, where agents smooth their consumption based on the historical evolution of income.

**Proposed economic model**

This section formulates an intertemporal consumption model based on the hypothesis that current consumption decisions depend not only on current income but also on past income. This historical dependence is incorporated through a linear Volterra convolution integral operator, where income is treated as a known function.

**Formulation of the model**

Let us consider that consumption $C(t)$ at time $t$ is determined by a weighted combination of past income $Y(s)$. This can be described by an equation of the type shown in **(2)**. Specifically, in the form:

$$C(t) = \int_0^t K\,(t-s)\,Y(s)\,ds, \qquad (3)$$

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where $C(t)$ is the consumption function at time $t$, $Y(s)$ is the income function at a past time $s \leq t$, and $K(t-s)$ is a kernel function representing the influence of past income on current consumption. This model captures the idea that individuals do not adjust their consumption instantaneously in response to current income, but rather do so by taking their recent experience into account. Depending on the choice of $K(u)$, varying degrees of memory or persistence in consumer behavior can be represented.

**Kernel selection**

The kernel function $K(t-s)$ is chosen to reflect how the past influences the present. Two common cases are: exponential memory (decay) and constant memory (moving average).

On the one hand, the *exponential memory* is given by the equation:

$$K(t-s) = \alpha e^{-\alpha(t-s)}, \quad \alpha > 0. \qquad (4)$$

Specifically, this kernel assigns greater weight to recent income and decreasing weight to past income. In this sense, it is consistent with the idea of progressive forgetting. On the other hand, the *constant memory* is expressed as follows:

$$K(t-s) = \begin{cases} \frac{1}{T}, & 0 \leq t-s \leq T, \\ 0, & t-s > T, \end{cases} \qquad T > 0. \qquad (5)$$

This kernel considers a time interval $T$ in which income carries equal weight in consumption. For the present study, the exponential kernel will be used due to its analytical tractability and because it adequately represents the decaying nature of economic memory.

**Exogenous income**

To simplify the analysis, it will be assumed that income is a continuous and known function, for example:

$$Y(t) = Y_0 e^{\beta t}, \qquad (6)$$

where $Y_0$ is the initial income and $\beta$ represents a growth rate (positive or negative). This assumption allows for the derivation of explicit or approximate solutions to the model and for the analysis of how consumption varies under different growth or recession scenarios.

**Economic interpretation**

This model has a clear interpretation: consumption is a weighted accumulation of past income, which can be linked to theories such as Friedman's (1957) permanent income hypothesis or the life-cycle

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hypothesis of Modigliani and Brumberg (1954). In both cases, consumption responds more slowly to short-term changes in income, and this inertia can be represented by a memory function.

**Model solution**

To solve the Volterra integral equation describing intertemporal consumption with memory, direct integration is applied. The exponential kernel and the assumed income function allow an exact analytical solution to be obtained without requiring approximation methods. Gripenberg et al. (1990).

**Base equation**

Specifically, the equation to be solved is:

$$C(t) = \int_0^t K\,(t-s)\,Y(s)\,ds, \qquad (7)$$

for the following defined functions: exponential kernel, $K(t-s) = \alpha e^{-\alpha(t-s)}$, and *exogenous income*, $Y(s) = Y_0 e^{\beta s}$. Where, the equation then transforms into:

$$C(t) = \int_0^t \alpha\, e^{-\alpha(t-s)} Y_0 e^{\beta s} ds. \qquad (8)$$

**Exact solution of the linear case**

Considering the exponential memory kernel $K(t-s) = \alpha e^{-\alpha(t-s)}$ and the exponential income function $Y(s) = Y_0 e^{\beta s}$, where $\alpha > 0$, $Y_0 > 0$, and $\beta \in \mathbb{R}$, the intertemporal consumption model can be solved analytically through direct integration.

Substituting these expressions into the Volterra integral formulation yields

$$C(t) = \int_0^t \alpha\, e^{-\alpha(t-s)} Y_0 e^{\beta s}\, ds. \qquad (9)$$

Since $\alpha$ and $Y_0$ are constant with respect to the integration variable $s$, the expression becomes

$$C(t) = \alpha Y_0 \int_0^t e^{-\alpha(t-s)}\, e^{\beta s}\, ds. \qquad (10)$$

At last, the exact analytical solution is obtained:

$$C(t) = \frac{\alpha Y_0}{\alpha+\beta}\left(e^{\beta t} - e^{-\alpha t}\right), \qquad \alpha + \beta \neq 0. \qquad (11)$$

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The restriction $\alpha + \beta \neq 0$ arises from the denominator of the analytical expression. However, the case $\alpha + \beta = 0$ must also be considered.

When $\beta = -\alpha$, the integral reduces to

$$C(t) \quad = \alpha Y_0 e^{-\alpha t} \int_0^t e^{0s}\, ds. \qquad (12)$$

Therefore, the solution for this particular case is

$$C(t) = \alpha Y_0 t e^{-\alpha t}, \qquad \alpha + \beta = 0. \qquad (13)$$

Thus, the consumption model admits an exact analytical solution for all real values of $\beta$, provided that the corresponding expression is used when $\alpha + \beta = 0$.

**Analysis of the result**

The analytical solution obtained in Equation **(11)** is valid for $\alpha + \beta \neq 0$, assuming $\alpha > 0$ and $Y_0 > 0$. The sign of $\alpha + \beta$ determines the relationship between the income growth rate and the decay rate of the memory kernel.

The following cases are distinguished:

- Case $\alpha + \beta > 0$: This condition implies $\beta > -\alpha$. If $\beta > 0$, consumption grows without converging to a finite equilibrium. However, the consumption-to-income ratio approaches $\alpha/(\alpha + \beta)$. If $\beta = 0$, consumption converges to $Y_0$, whereas if $-\alpha < \beta < 0$, consumption converges to zero.
- Case $\alpha + \beta = 0$: This condition implies $\beta = -\alpha$. The consumption function becomes $C(t) = \alpha Y_0 t e^{-\alpha t}$. Since $\alpha > 0$, consumption converges to zero as $t \to \infty$.
- Case $\alpha + \beta < 0$: This condition implies $\beta < -\alpha$. Income decreases more rapidly than the memory kernel decays. Consequently, consumption remains nonnegative and converges to zero as $t \to \infty$.

From an economic perspective, the parameter $\alpha$ represents the rate at which the influence of historical income decreases, whereas $\beta$ describes the growth or decline of income.

The sum $\alpha + \beta$ reflects the interaction between these two rates within the integral formulation. Nevertheless, the long-term behavior of consumption is primarily determined by the sign of $\beta$, while $\alpha$ controls the persistence of historical income and the speed of consumption adjustment.

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## MATERIALS AND METHODS

The methodology adopted in this research focused on the development and analysis of a dynamic economic model that incorporates the concept of memory through the use of Volterra integral equations. The following steps were followed to carry out this study: Development of the theoretical model, Computational implementation, Simulation and visualization and Validation of results.

First, the consumption model was formulated using an exponential memory kernel and an exogenous income function. An exact analytical solution was obtained through direct integration, allowing the relationship between the memory parameter $\alpha$ and the income growth rate $\beta$ to be examined.

Second, the model was implemented computationally in Python. For the numerical approximation, the Volterra integral was represented by a discrete convolution of the income series and the exponential memory kernel:

$$C(t_n) \approx \Delta t \sum_{j=0}^{n} \alpha\, e^{-\alpha(t_n - t_j)} Y(t_j), \qquad (14)$$

where $t_n = n\Delta t$, $\Delta t$ represents the time step, and $Y(t_j)$ denotes income at time $t_j$.

Third, consumption trajectories were examined over a simulation horizon of 360 monthly periods. Different values of the memory parameter were considered to analyze their influence on the adjustment of consumption. In particular, the values $\alpha = 0.03$, $\alpha = 0.08$, and $\alpha = 0.15$ were used for the comparison presented in Figure 1.

Finally, the analytical and numerical results were interpreted in terms of consumption adjustment and the persistence of historical income. Additional illustrative scenarios involving inflation, savings, and debt were examined through the corresponding graphical representations.

## RESULTS AND DISCUSSION

### Results

The solution obtained for the intertemporal consumption model with memory:

$$C(t) = \frac{\alpha Y_0}{\alpha + \beta}\left(e^{\beta t} - e^{-\alpha t}\right), \qquad \alpha + \beta \neq 0. \qquad (15)$$

shows how consumption depends on income history, weighted by an exponential memory function. This relationship reflects the fact that individuals do not respond immediately to changes in income,

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but rather their consumption gradually adjusts based on their past experience, which is consistent with established economic theories such as the permanent income theory and the life cycle hypothesis.

Therefore, if $\beta > 0$, income and consumption increase over time. Although consumption does not converge to a fixed equilibrium level, the consumption-to-income ratio approaches $\alpha/(\alpha + \beta)$.

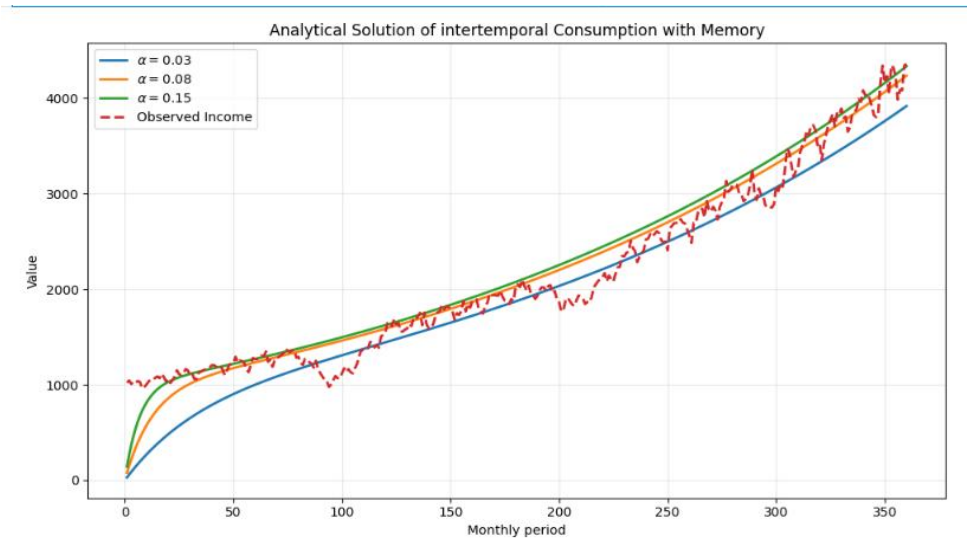
**Interpretation of the parameters**

On one hand, the parameter $\alpha$ controls the depth of memory. Specifically, the higher the $\alpha$, the faster the agent forgets past income, and therefore consumption adjusts more quickly to current income. On the other hand, the parameter $\beta$ represents the income growth rate. Therefore, if $\beta > 0$, income and consumption increase over time, without consumption converging to a finite equilibrium level. This behavior is consistent with the idea that consumers may smooth consumption in response to temporary income fluctuations, while adjusting their consumption trajectories to persistent changes in income.

**Computational implementation of the model in Python**

The following figure illustrates the analytical solution for intertemporal consumption with memory, evaluated using Python. The evolution of consumption $C(t)$ over time is plotted, considering different values of the memory parameter.

Figure 1:
*Consumption model with memory (explicit function)*



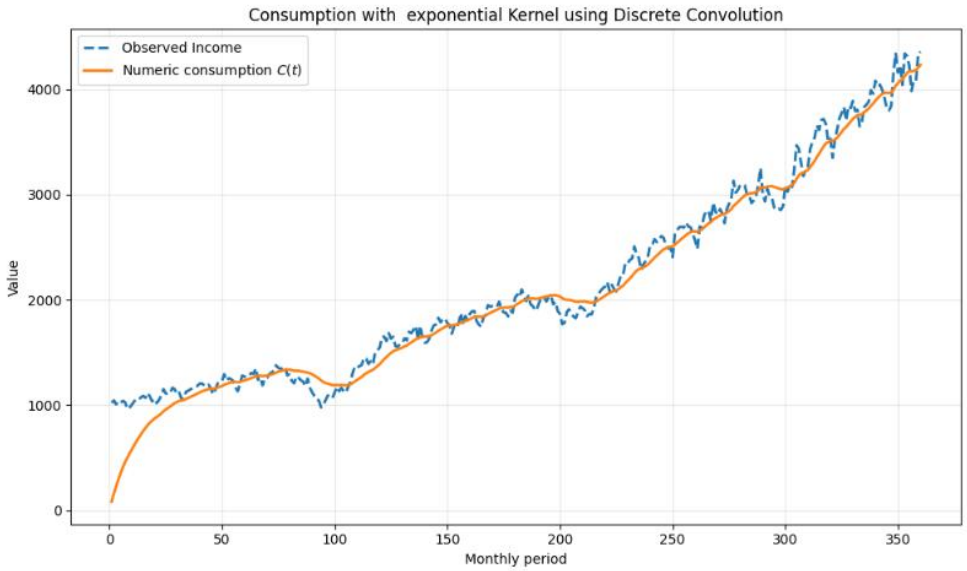
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**Numerical solution using convolution**

The consumption $C(t)$ can also be approximated numerically by convoluting the kernel function $K(t-s)$ and the income $Y(s)$. The following graphically shows a numerical solution using discrete integration.

**Figure 2:**

*Consumption with exponential kernel (accelerated growth)*




This code offers a direct approach to calculating consumption based on the Volterra equation, using a numerical approach that simulates the behavior of the integral.

**Savings and debt models with variable interest rates**

***Interest rate on savings and debt***

On the one hand, if you have savings, then they accrue interest. Whereas, if you are in debt, then you pay interest (negative rate).

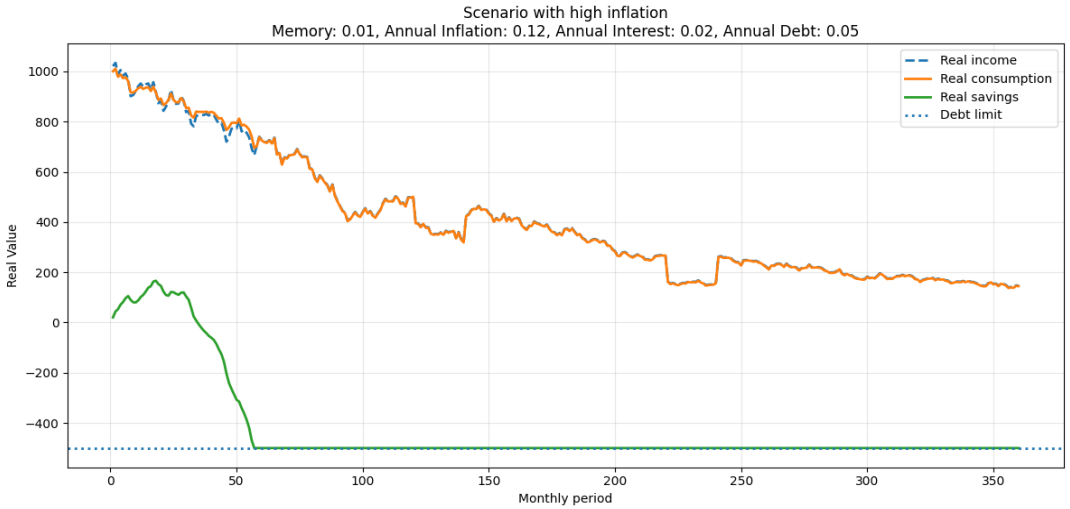
***Inflation***

With respect to inflation, income and consumption are adjusted by a rising price index. This simulates an economy where the value of money changes over time. This situation is illustrated in the graph in the following Figure 3

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**Figure 3:**

*High inflation*

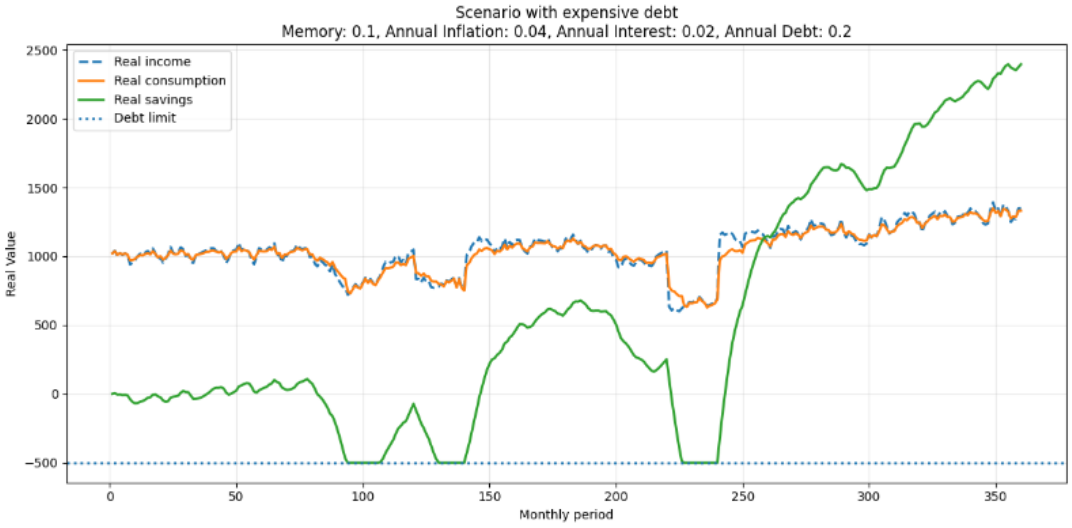



***Costly debt***

Within this context, the agent is strained between maintaining their consumption based on past habits and avoiding the high cost of borrowing. This creates a forced equilibrium. Figure 3.2 illustrates this situation.

**Figure 4:**

*Costly Debt*



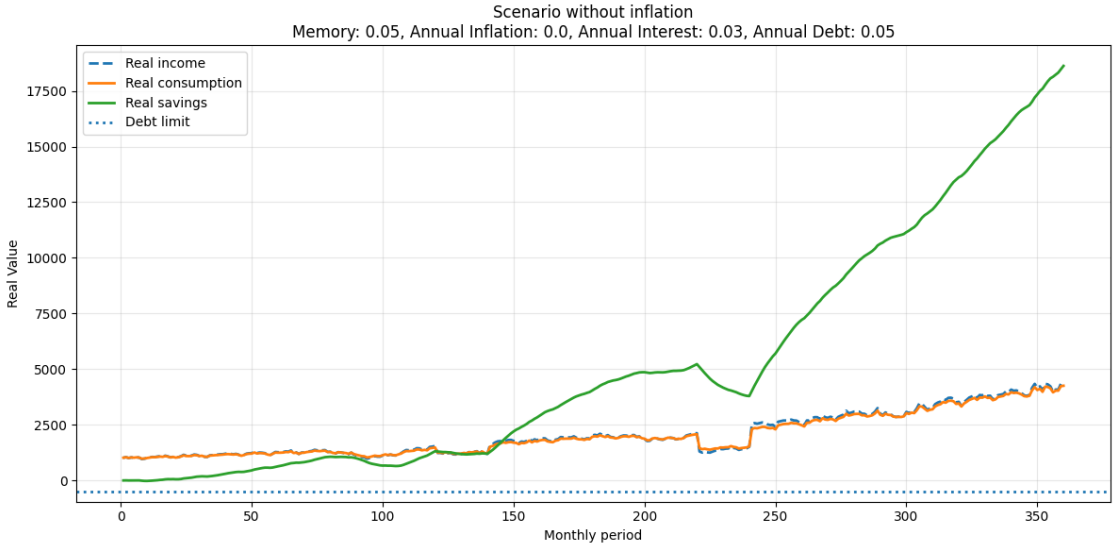
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***Without inflation***

Under these circumstances, the agent can save without worrying about inflation or excessive debt.

**Figure 5:**

*without inflation*




**Interpretation of results**

**Consumption model with memory**

Figure 1 shows the evolution of consumption $C(t)$ over 360 monthly periods, considering three values of the memory parameter: $\alpha = 0.03$, $\alpha = 0.08$, and $\alpha = 0.15$.

The consumption model is represented by the analytical expression:

$$C(t) = \frac{\alpha Y_0}{\alpha + \beta}\left(e^{\beta t} - e^{-\alpha t}\right), \qquad \alpha + \beta \neq 0. \qquad (16)$$

In this expression, $Y_0$ denotes the initial income, $\alpha$ represents the memory decay parameter, $\beta$ is the income growth rate, and $t$ denotes time.

Regarding the interpretation of the graph, the following observations can be made:

- Shape of the curves: The three consumption trajectories exhibit an increasing trend throughout the simulated period. Their differences reflect the influence of the memory parameter on consumption adjustment.

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- Initial adjustment: At the beginning of the simulation, consumption increases gradually as historical income contributions accumulate. Higher values of $\alpha$ produce a faster initial adjustment.
- Memory effect: For $\alpha = 0.03$, consumption responds more slowly to income variations because past income retains its influence for a longer period. For $\alpha = 0.08$ and $\alpha = 0.15$, the adjustment becomes progressively faster.
- Long-term behavior: The consumption trajectories do not converge to a fixed equilibrium value within the simulated period. Instead, they follow the general upward trend of the income series.

Therefore, Figure 1 illustrates how different memory parameters affect the speed of consumption adjustment. Larger values of $\alpha$ reduce the persistence of historical income, allowing consumption to respond more rapidly to recent income variations.

**Consumption with the exponential kernel (accelerated growth)**

Figure 2 shows the evolution of intertemporal consumption with memory over 360 monthly periods, obtained through discrete convolution using an exponential memory kernel.

The theoretical income function considered in the analytical model is expressed as:

$$Y(t) = Y_0 e^{\beta t}. \qquad (17)$$

For the numerical implementation, the observed income series is incorporated into the convolution with the exponential memory kernel, allowing consumption to depend on current and historical income.

Regarding the interpretation of the graph, the following observations can be made:

- Growth of consumption: The numerical consumption trajectory exhibits an increasing trend throughout the simulated period, following the general evolution of observed income.
- Memory effect ($\alpha$): The parameter $\alpha$ determines the rate at which the influence of historical income decreases. Smaller values of $\alpha$ imply greater memory persistence, whereas larger values produce a faster adjustment to recent income changes.
- Relationship with income ($Y$): Consumption follows the overall trend of observed income, although its short-term variations are less pronounced. This behavior reflects the weighted contribution of historical income through the exponential kernel.

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- Smoothness of the curve: The consumption trajectory is smoother than the observed income series. This illustrates the smoothing effect of discrete convolution and the gradual adjustment of consumption to income fluctuations.

Therefore, Figure 2 illustrates how the exponential memory kernel incorporates historical income into current consumption. The resulting trajectory reflects both the upward trend of income and the smoothing effect associated with economic memory.

**Interest rate on savings and debt**

The graphs in Figures 3, 4 and 5 represent the evolution of real income, consumption and savings under different illustrative economic scenarios, considering memory effects, interest rates, debt and inflation.

***Scenario with high inflation***

The following is a detailed analysis of parameters, real income, consumption, and savings under the simulated conditions:

- Parameters: Memory decay parameter $\alpha = 0.01$ (high memory persistence), Interest = $0.02$, Debt Interest = $0.05$, Inflation = $0.04$ (high).
- Real income: Decreases over time due to the loss of purchasing power caused by inflation.
- Consumption: Decreases progressively in the simulated scenario as real income deteriorates.
- Savings: Falls rapidly, reaching the debt limit represented in the simulation.

***Scenario with costly debt***

- Parameters: Memory = $0.1$ (high), Interest = $0.02$, Debt Interest = $0.09$ (high), Inflation = $0.01$.
- Consumption: It remains more stable due to the high memory, which induces the agent to maintain its lifestyle.
- Savings: Initially decreases, but soon stabilizes due to the high cost of debt.
- The agent avoids excessive debt by forcibly adjusting their consumption.

***No-inflation scenario***

- Parameters: Memory = $0.05$, Interest = $0.05$, Debt Interest = $0.05$, Inflation = $0.00$.
- Real income: Remains stable, allowing for better planning of consumption.
- Consumption: It is more sustained and balanced.

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- Savings: It grows steadily, since there is no loss of purchasing power and real interest is gained.

***Comparative table of interest rates on savings and debt***

Table **(1)** below shows how different economic contexts, such as high inflation or costly debt, are associated with changes in consumption and savings within the simulated scenarios.

**Table 1:**

*Comparison of interest rates on savings and debt*

| Scenario | Main risk | Main behavior | Result |
|---|---|---|---|
| High Inflation | Loss of purchasing power | Reduction in savings and greater debt exposure | Declining consumption and savings |
| Costly Debt | High interest rates on debt | Consumption adjustment under borrowing costs | Relative stabilization |
| Without Inflation | Absence of inflationary pressure | More stable consumption and increasing savings | Favorable simulated conditions |

The results presented in Table **(1)** suggest that inflation may reduce purchasing power and place pressure on savings. When borrowing costs are higher, consumption adjustment becomes relevant to the simulated trajectory. In the absence of inflation, the scenario shows more stable consumption and an increase in savings.

Regarding the implications for economic policy, these types of models may provide a complementary framework for examining how changes in income affect consumption over time. For example, a temporary increase in income may influence consumption beyond the period in which the increase occurs, depending on the persistence of the memory kernel.

In scenarios of sustained income growth, models with memory may help describe the evolution of consumption over longer periods. However, their application to aggregate demand or policy evaluation would require additional assumptions and empirical validation.

Furthermore, this formulation could be extended to examine social programs in which previous income transfers influence subsequent consumption decisions.

On the other hand, regarding the comparison with memoryless models, unlike formulations where $C(t) = cY(t)$, the proposed model incorporates historical income through a memory kernel. In a memoryless model, consumption responds directly to contemporaneous income, whereas the Volterra formulation distributes the influence of income over time.

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## CONCLUSIONS

The study highlights the following considerations related to scientific value: consumption with memory exhibits a gradual adjustment process, whose long-term behavior depends on the memory parameter $\alpha$ and the income growth rate $\beta$. When $\beta > 0$, consumption increases over time without converging to a finite equilibrium; when $\beta = 0$, it approaches the constant income level $Y_0$; and when $\beta < 0$, consumption tends to zero. Economic memory also influences consumption dynamics, since greater memory persistence allows past income to affect current consumption for a longer period.

Likewise, the simulated scenarios suggest that high inflation may reduce real income and place pressure on consumption and savings. In the scenario with costly debt, higher borrowing costs are associated with adjustments in consumption and savings. The scenario without inflation shows more stable consumption and increasing savings under the assumed conditions. These results illustrate possible economic responses within the model and should not be interpreted as general empirical conclusions.

With respect to the theoretical and mathematical contributions, the following can be mentioned: first, a dynamic mathematical model of consumption was developed using a Volterra integral formulation that incorporates memory, allowing current consumption to depend on a weighted accumulation of historical income; second, consumption adjusts gradually over time, with its behavior determined by the interaction between the memory parameter and the income growth rate. For $\alpha + \beta \neq 0$, the analytical solution is given by:

$$C(t) = \frac{\alpha Y_0}{\alpha + \beta}\left(e^{\beta t} - e^{-\alpha t}\right).$$

For $\beta > 0$, the consumption-to-income ratio converges to a constant:

$$\lim_{t\to\infty} \frac{C(t)}{Y(t)} = \frac{\alpha}{\alpha + \beta}.$$

In contrast, when $\beta = 0$, consumption approaches $Y_0$, while for $\beta < 0$, consumption tends to zero.

Third, regarding the role of the parameters, $\alpha$ controls the rate at which the influence of past income decreases, with higher values implying lower memory persistence. The parameter $\beta$ represents the income growth rate and determines whether consumption increases or decreases in the long term. Fourth, the results are consistent with the idea of gradual consumption adjustment considered in the permanent income and life-cycle approaches, although the proposed formulation does not establish a direct equivalence with these theories. Fifth, regarding economic policy, the model provides a mathematical framework that could be extended to examine the effects of temporary income changes

and social transfers, subject to further empirical validation. Sixth, the computational implementation included the analytical solution and a numerical approximation using discrete convolution, illustrating how consumption evolves under different memory parameters. Finally, the inflation, savings, and debt scenarios show different consumption and savings trajectories under the assumed economic conditions. These simulations provide an initial illustration of the model's potential applications, while further specification and empirical analysis are required to evaluate its performance in real economic settings.

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